

Diquark Condensates and Compact Star Cooling

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ABSTRACT

The effect of color superconductivity on the cooling of quark stars and neutron stars with large quark cores is investigated. Various known and new quark-neutrino processes are studied. As a result, stars being in the color flavor locked (CFL) color superconducting phase cool down extremely fast. Quark stars with no crust cool down too rapidly in disagreement with X-ray data. The cooling of stars being in the $N_f = 2$ color superconducting (2SC) phase with a crust is compatible with existing X-ray data. Also the cooling history of stars with hypothetical pion condensate nuclei and a crust does not contradict the data.

Subject headings: stars: evolution, stars: neutron, pulsars: general

1. Introduction

The interiors of compact stars have been discussed as systems where high-density phases of strongly interacting matter do occur in nature, see (Glendenning 1996) and (Weber 1999). The consequences of different phase transition scenarios for the cooling behaviour of compact stars have been reviewed recently (Page 1992; Schaab et al. 1996, 1997) in comparison with existing X-ray data.

A particular discussion has been devoted to the idea that a strong attraction in three flavor *uds*-quark matter may allow for the existence of super-dense anomalous nuclei and strange quark stars (Bodmer 1971; Witten 1984; de Rujula and Glashow 1984). Thereby, in dependence on the value of the bag constant B different possible types of stars were discussed: ordinary neutron stars (NS) without any quark core, neutron stars with quark matter present only in their deep interiors (for somewhat intermediate values of B), neutron stars with a large quark core (QCNS) and a crust typical for neutron stars, and quark stars (QS) with a tiny crust of normal matter and with no crust (both for low B values). By QCNS we un-

derstand compact stars in which the hadronic shell is rather narrow in the sense that it does not essentially affect the cooling which is mainly due to the quark core. However, this hadronic shell plays the role of an insulating layer between quark matter and the normal crust. By QS we mean compact stars for which the hadronic shell is absent what allows only for tiny crusts with maximum densities below the neutron drip density ($\sim 10^{11}$ g/cm³) and masses $M_{\text{cr}} \lesssim 10^{-5} M_{\odot}$ (Alcock et al. 1986; Horvath et al. 1991). Therefore, we suppose that the difference between the models of QS and QCNS regarding their cooling behaviour is only in the thickness of the crust. This results in completely different relations between internal (T) and surface (T_s) temperatures for QS and QCNS.

Recent works (Alford et al. 1998; Rapp et al. 1998; Schäfer 1998; Carter and Diakonov 1999; Rapp et al. 1999; Blaschke and Roberts 1998; Bloch et al. 1999; Pisarski and Rischke 1999; Alford et al. 1999; Schäfer and Wilczek 1999a; Alford et al. 1999; Schäfer and Wilczek 1999b) demonstrate the possibility of diquark condensates characterized by large pairing gaps ($\Delta \sim 100$ MeV) in quark cores of some neutron stars and in QS

and discuss different possible phases of quark matter. Large gaps arise for quark-quark interactions motivated by instantons (Diakonov et al. 1996; Carter and Diakonov 1999; Rapp et al. 1999) and by nonperturbative gluon propagators (Blaschke and Roberts 1998; Bloch et al. 1999; Pisarski and Rischke 1999).

To be specific in our predictions we will consider models of the canonical QS and QCNS of 1.4 solar masses ($1.4 M_\odot$) at a constant density. The constant density profile is actually a very good approximation for QS of the mass $M \leq 1.4 M_\odot$, see Alcock et al. (1986). We consider the model of QCNS with a crust of the mass $M_{\text{cr}} \sim 10^{-1} M_\odot$, the model of QS with a tiny crust mass ($M_{\text{cr}} \lesssim 10^{-5} M_\odot$) and the model of QS with no crust. For QCNS we shall use the same T_s/T ratio as for ordinary NS, see (Tsuruta 1979; Maxwell 1979; Shapiro and Teukolsky 1983), whereas for QS we use somewhat larger values for this ratio, namely $T_s = 5 \cdot 10^{-2} T$ for a tiny crust (Horvath et al. 1991) and $T_s = T$ for a negligible crust (Pizzochero 1991). In the latter case, however, we assume the existence of black body photon radiation from the surface as for the cases of more extended crusts. We will estimate the cooling of QS and QCNS first in absence of color superconductivity and then in presence of color superconductivity for small quark gaps ($\Delta \sim 0.1 \dots 1$ MeV), as suggested by Bailin and Love (1984) and for large gaps ($\Delta \sim 100$ MeV) as obtained in Refs. (Alford et al. 1998; Rapp et al. 1998; Schäfer 1998; Carter and Diakonov 1999; Rapp et al. 1999; Blaschke and Roberts 1998; Pisarski and Rischke 1999; Alford et al. 1999; Schäfer and Wilczek 1999a; Alford et al. 1999; Schäfer and Wilczek 1999b). In the latter case we will consider two phases: the color-flavor-locked *uds*-phase (Alford et al. 1999; Schäfer and Wilczek 1999a; Alford et al. 1999; Schäfer and Wilczek 1999b) and the $N_f = 2$ color superconducting phase (Alford et al. 1998; Rapp et al. 1998; Schäfer 1998; Blaschke and Roberts 1998; Carter and Diakonov 1999; Rapp et al. 1999; Bloch et al. 1999; Pisarski and Rischke 1999), in which the *s*-quark is absent and the *ud*-diquark condensate selects a direction in color space whereby the color charge has to be compensated by the remaining unpaired quarks.

Finally, we want to discuss the question whether the hypothesis of a color superconducting quark

matter phase in compact star interiors is compatible with existing X-ray data.

2. Normal quark matter

A detailed discussion of the neutrino emissivity of quark matter has first been given by Iwamoto (1982) where the possibility of color superconductivity has not been discussed. In this work the quark direct Urca reactions (QDU) $d \rightarrow ue\bar{\nu}$ and $ue \rightarrow d\nu$ were suggested as the most efficient processes. Their emissivities were estimated as ¹

$$\epsilon_\nu^{\text{QDU}} \sim 10^{26} \alpha_s \left(\frac{\rho_b}{\rho_0} \right) Y_e^{1/3} T_9^6 \text{ erg cm}^{-3} \text{ sec}^{-1}, \quad (1)$$

where at baryon densities $\rho_b \simeq 2\rho_0$ the strong coupling constant is $\alpha_s \approx 1$ and decreases logarithmically at still higher densities (Kisslinger and Morley 1976). The nuclear saturation density is $\rho_0 = 0.17 \text{ fm}^{-3}$, $Y_e = \rho_e/\rho_b$ is the electron fraction, and T_9 is the temperature in units of 10^9 K. The larger the density of the *uds*-system, the smaller is its electron fraction. For a density $\rho_b \sim 3\rho_0$ one can expect a rather low electron fraction of strange quark matter $Y_e \sim 10^{-5}$ (Glendenning 1996) and eq. (1) yields $\epsilon_\nu^{\text{QDU}} \sim 10^{25} T_9^6 \text{ erg cm}^{-3} \text{ sec}^{-1}$, see (Duncan 1983; Horvath et al. 1991). We did not yet discuss the strange quark contribution given by the direct Urca processes $s \rightarrow ue\bar{\nu}$ and $ue \rightarrow s\nu$. Although these processes can occur, their contribution is suppressed compared to the corresponding *ud*-reactions (Duncan 1983) by an extra factor $\sin^2 \theta_C \sim 10^{-3}$, where θ_C is the Cabibbo angle.

If for somewhat larger density the electron fraction was too small ($Y_e < Y_{ec} \simeq \sqrt{3}\pi m_e^3 / (8\alpha_s^{3/2} \rho_b) \leq 2 \cdot 10^{-8}$, for $\alpha_s \simeq 0.7$ and $\rho_b \simeq 5\rho_0$, m_e is the electron mass), then all the QDU processes would be completely switched off (Duncan 1983) and the neutrino emission would be governed by two-quark reactions like the quark modified Urca (QMU) $dq \rightarrow uqe\bar{\nu}$ and the quark bremsstrahlung (QB) processes $q_1 q_2 \rightarrow q_1 q_2 \nu \bar{\nu}$. The emissivities of the QMU and QB processes were estimated as (Iwamoto 1982)

$$\epsilon_\nu^{\text{QMU}} \sim \epsilon_\nu^{\text{QB}}(\text{sqr}) \sim 10^{20} T_9^8 \text{ erg cm}^{-3} \text{ sec}^{-1}. \quad (2)$$

¹We note that the numerical factors in the estimates below are valid only within an order of magnitude. This is sufficient for the qualitative comparison of cooling scenarios we present in this work.

The latter estimate of QB emissivity is done in the suggestion that the exchanged gluon is screened. If one suggests that quarks are coupled by transverse non-screened gluons then one gets (Price 1980) $\epsilon_\nu^{\text{QB}}(\text{unscr}) \sim 10^{22} T_9^6 \text{ erg cm}^{-3} \text{ sec}^{-1}$. With a nonperturbative gluon exchange (Blaschke and Roberts 1998; Bloch et al. 1999) we expect that the estimate $\epsilon_\nu^{\text{QB}}(\text{scr})$ is more appropriate than the one given by $\epsilon_\nu^{\text{QB}}(\text{unscr})$. Therefore, in evaluating the emissivity of QB processes we will use (2). Other neutrino processes (like the plasmon decay $\gamma_{\text{pl}} \rightarrow ee^{-1} \rightarrow \nu\bar{\nu}$ which goes via coupling to intermediate electron-electron hole states, and the corresponding color plasmon decay $g_{\text{pl}} \rightarrow qq^{-1} \rightarrow \nu\bar{\nu}$ which goes via coupling of the gluon to quark-quark hole states (Iwamoto 1982), see Fig. 4) have much smaller emissivities in the normal quark matter phase under consideration and can be neglected.

Among the processes in the crust, the electron bremsstrahlung on nuclei gives the largest contribution to the emissivity as estimated in (Iwamoto 1982)

$$\epsilon_\nu^{\text{cr}} \sim 10^{21} \left(\frac{M_{\text{cr}}}{M_\odot} \right) T_9^6 \text{ erg cm}^{-3} \text{ sec}^{-1}. \quad (3)$$

This contribution can be neglected for QS due to a tiny mass of the QS crust $M_{\text{cr}} \lesssim 10^{-5} M_\odot$.

Besides, one should add the photon contribution

$$\epsilon_\gamma \simeq 2 \cdot 10^{18} \left(\frac{R}{10 \text{ km}} \right)^2 T_{s7}^4 \text{ erg cm}^{-3} \text{ sec}^{-1}, \quad (4)$$

where T_{s7} is the surface temperature in units of 10^7 K , see (Shapiro and Teukolsky 1983). This process becomes the dominant one for QS at essentially shorter times than for the QCNS due to the higher T_s/T ratios for the former.

Internal and surface temperatures are related by a coefficient determined by the scattering processes occurring in the outer region, where the electrons become non-degenerate. For NS with rather thick crust, an appropriate fit to numerical calculations (Tsuruta 1979) is given by a simple formula (Shapiro and Teukolsky 1983)

$$T_s = (10 T)^{2/3}, \quad (5)$$

where T_s and T both are measured in units of K. We shall use this expression when dealing with

QCNS. A rough estimate of (5) yields $T_s = a \times 10^{-2} T$ with $a \simeq 0.2 - 2$ in dependence on the value of the internal temperature varying in the interval $10^{10} \dots 10^7 \text{ K}$ of our interest. In the QS the crust is much more thin than in the NS and the ratio T_s/T should be significantly larger. Therefore, depending on the thickness of the crust we shall use two estimates for QS scenarios: $T_s = 5 \times 10^{-2} T$ for a tiny crust (Horvath et al. 1991), and $T_s \simeq T$ for negligible crust (Pizzochero 1991).

In order to compute the cooling history of the star we still need the specific heat of the electron, photon, gluon and quark sub-systems. In accordance with the estimates of Iwamoto (1982), Horvath et al. (1991) we have

$$c_e \simeq 0.6 \cdot 10^{20} \left(\frac{Y_e \rho_b}{\rho_0} \right)^{2/3} T_9 \text{ erg cm}^{-3} \text{ K}^{-1}, \quad (6)$$

$$c_\gamma \simeq 0.3 \cdot 10^{14} T_9^3 \text{ erg cm}^{-3} \text{ K}^{-1}, \quad (7)$$

$$c_g \simeq 0.3 \cdot 10^{14} N_g T_9^3 \text{ erg cm}^{-3} \text{ K}^{-1}, \quad (8)$$

$$c_q \simeq 10^{21} \left(\frac{\rho_b}{\rho_0} \right)^{2/3} T_9 \text{ erg cm}^{-3} \text{ K}^{-1}, \quad (9)$$

where N_g is the number of different color states of massless gluons. The very small contribution to the specific heat of the crust can be neglected (Lattimer et al. 1994). The cooling equation reads

$$\sum_{i=q,e,\gamma,g} c_i \cdot \frac{dT}{dt} = -\epsilon_\gamma - \sum_{j=\text{QDU,QMU,QB,cr}} \epsilon_\nu^j, \quad (10)$$

where the summation is over all contributions to the specific heats and emissivities as discussed above. The evolution at large times on which we are focusing our interest here is insensitive to the assumed value of the initial temperature T_0 . We checked it using different values of initial temperature. To be specific we choose the value $T_{0,9} = 10$ as a typical initial temperature for proto-neutron stars.

In Figs. 1-3 we show the cooling history of QCNS with standard thickness of the NS crust (when internal and surface temperatures are related by Eq. (5)), QS with a tiny crust ($T_s = 5 \cdot 10^{-2} T$) and QS with negligible crust ($T_s = T$), respectively. Solid curves are for the matter suggested to be in the normal state, i. e. in the absence of color superconductivity. Different groups of data points are taken from Table 3 of Ref.

(Schaab et al. 1999) where the notations are explained. In the lower panels of Figs. 1-3, we show the results for $Y_e > Y_{ec}$ taking $Y_e = 10^{-5}$, $\alpha_s = 1$, $\rho = 3\rho_0$. This is a representative set of parameters for which the QDU processes contribute to the cooling. We see that two low-temperature pulsars can be explained as QCNS being in normal state (Fig. 1, thick solid curve, lower panel) and many observations can be interpreted as QS in normal state with a tiny crust (Fig. 2, lower panel).

The upper panels of Figs. 1-3 demonstrate the cooling history of QCNS and QS for $Y_e < Y_{ec}$, namely for $Y_e = 0$, $\alpha_s = 0.7$, and $\rho = 5\rho_0$. QCNS being in normal state (Fig. 1, thick solid curve, upper panel) cool down rather slowly but are still in agreement with the data for a few pulsars. For QS with a tiny crust (Fig. 2, $T_e = 5 \times 10^{-2}T$) we get a nice fit of many data points.

In both cases $Y_e < Y_{ec}$ and $Y_e > Y_{ec}$, see Fig. 3, QS with negligible crust cool down too fast in disagreement with the X-ray data.

3. Color superconductivity

In the standard scenario of NS cooling the inclusion of nucleon pairing suppresses the emissivity resulting in a more moderate cooling. Now, considering QS and QCNS we will show that we deal with the opposite case.

Due to the pairing, the emissivity of QDU processes is suppressed by a factor $\exp(-\Delta/T)$ and the emissivities of QMU and QB processes are suppressed by a factor $\exp(-2\Delta/T)$ for $T < T_c$. Thereby in our calculations we will use expression (1) for QDU suppressing the rate by $\exp(-\Delta/T)$ and expressions (2) for QMU and QB suppressing the rates by $\exp(-2\Delta/T)$ for $T < T_c$. We also observe that plasmon and color plasmon decay processes are switched off in the superconducting phase when the photons and the gluons acquire masses due to the Higgs effect, as it happens for photons in usual superconductors. Voskresensky et al. (1998) however demonstrated that in superconducting matter there appears a new neutrino neutral current process analogous to the plasmon decay but now due to a massive photon decay. Its emissivity is suppressed by the factor $\exp(-m_\gamma/T)$ rather than by $\exp(-\Delta/T)$, as for direct Urca processes, or by $\exp(-2\Delta/T)$, as for modified Urca and corresponding bremsstrahlung

processes. This results in a large contribution for small but finite values of m_γ . Naively, one could expect that in a color superconductor the squared photon mass is proportional to the fine structure constant $\alpha = 1/137$ as it is in ordinary superconductors. Then it would be much smaller than the squared gluon mass since the latter quantity in the color superconducting phase has to be proportional to the corresponding strong coupling constant α_s . In reality, due to the common gauge transformation for electromagnetic and color fields one deals with mixed photon-gluon excitations. We demonstrate this using the expression for the free energy density of the color superconducting phase (Bailin and Love 1984),

$$\begin{aligned} f = & f_n + ad^*d + \frac{1}{2}b(d^*\vec{d})^2 \\ & + c(\nabla + iq\vec{A} + \frac{ig\vec{B}}{\sqrt{3}})d^*(\nabla - iq\vec{A} - \frac{ig\vec{B}}{\sqrt{3}})d \\ & + \frac{(\text{rot}\vec{A})^2}{8\pi} + \frac{(\text{rot}\vec{B})^2}{8\pi}, \end{aligned} \quad (11)$$

where f_n is the normal part of the free energy density, $a = \mu p_{Fq}t/\pi^2$, $t = (T - T_c)/T_c < 0$, $b = 7\zeta(3)\mu p_{Fq}/(8\pi^4T_c^2)$, $c = p_{Fq}^2b/(6\mu^2)$, $\mu \simeq p_{Fq}$ is chemical potential of ultra-relativistic quark, $q = \sqrt{\alpha}/3$ is the electric charge of a ud -pair, $\alpha = 1/137$. We have introduced an interaction with two gauge fields A_μ and B_μ . A_μ is the electromagnetic field and B_μ relates to the gluons. For simplicity we consider only fluctuations of space-like components of the fields and assume the B_μ field to be an Abelian field. Variation of (11) with respect to the fields gives the corresponding equations of motion. Taking $d = d_0 + d'$, where $d_0 = \sqrt{-a/b}$ is the order parameter and d' is the fluctuating field, we linearize the equations of motion for the fluctuating fields d' , \vec{A} , \vec{B} . Solving these equations in the Fourier representation, we get three branches of the spectrum. The branch $\omega^2 = \vec{q}^2 + 2|\alpha|$ corresponds to fluctuations of the order parameter characterized by a large mass $m_d = \sqrt{-2t\mu p_{Fq}}/\pi \sim m_\pi = 140$ MeV. The branch

$$\omega^2 = \vec{q}^2 + m_{\gamma,g}^2 \quad (12)$$

describes a massive photon-gluon excitation with a mass $m_{\gamma,g}^2 = 8\pi c(\alpha + 3\alpha_s)d_0^2/9$. The extra branch $\omega^2 = \vec{q}^2$ describes a massless mixed photon-gluon Goldstone excitation.

Thus in difference with a usual proton superconducting phase of NS where the photon has rather low mass $m_\gamma = d_0 \sqrt{8\pi c \alpha} \simeq 4$ MeV for $\mu \simeq 400$ MeV, in the color superconductor we, probably, deal with a much more massive mixed photon-gluon excitation (with α being replaced by $\alpha + 3\alpha_s$) and with the corresponding Goldstone boson².

The Goldstone boson does not contribute to the mentioned photon-gluon decay process, whereas the massive excitation does. Now, armed with an expression for the photon-gluon mass we may estimate the emissivity of the corresponding processes $(\gamma - g) \rightarrow ee^{-1} + qq^{-1} \rightarrow \nu\bar{\nu}$, where e^{-1} and q^{-1} are the electron hole and the quark hole, respectively, see Fig. 4. Using the result for $\gamma \rightarrow ee^{-1} + pp^{-1} \rightarrow \nu\bar{\nu}$ (Voskresensky et al. 1998) we easily get

$$\begin{aligned} \epsilon_{(\gamma-g)} &\sim 10^{29} \left(\frac{m_{\gamma,g}}{\text{MeV}} \right)^{7/2} T_9^{3/2} \left(1 + \frac{3T}{2m_{\gamma,g}} \right) \\ &\times \exp(-m_{\gamma,g}/T) \text{ erg cm}^{-3} \text{ sec}^{-1}, (13) \end{aligned}$$

for $T < m_{\gamma,g}$, and using the condition $\Delta \ll \mu$. As we see, the emissivity of this process is strongly suppressed for the values $m_{\gamma,g} \simeq 70$ MeV following from eq. (12). Also the specific heat of this mixed photon-gluon excitation is suppressed by the same exponential factor $\exp(-m_{\gamma,g}/T)$. For the Goldstone excitation the contribution to the specific heat is given by Eq. (7).

For the quark specific heat at $T < T_c$ we use an expression similar to the one which applies for the case of nucleon pairing (Mühschlegel 1959; Maxwell 1979; Horvath et al. 1991), i.e.

$$\begin{aligned} c_{sq} &= 3.2 c_q (T_c/T) \exp(-\Delta/T) \\ &\times [2.5 - 1.7 T/T_c + 3.6 (T/T_c)^2], (14) \end{aligned}$$

where T_c is related to Δ as $\Delta = 1.76 T_c$ for the case of small gaps. For CFL and 2SC phases we will use $T_c \simeq 0.4 \Delta$. Actually, in the latter case the relation between T_c and Δ is unsettled. However, one believes that the coefficient in standard BCS formula $T_c \simeq 0.57 \Delta$ is appreciably reduced as impact

of instanton-anti-instanton molecules (Rapp et al. 1999). The mentioned uncertainty in the value of T_c for CFL and 2SC phases does not significantly affect the cooling curves since the dominant effect comes from the exponential factor where Δ enters rather than T_c .

Now, armed with all necessary expressions we may estimate the cooling of QS and QCNS being in *uds*- or 2SC phases (except for the crust).

3.1. Cooling of different *uds*-phases: *uds*-small gaps, CFL ($Y_e > Y_{ec}$), and CFL ($Y_e < Y_{ec}$)

We select the following values of the pairing gaps: small gaps $\Delta = 0.1 \dots 1$ MeV as suggested to occur in the color superconducting region by Bailin and Love (1984) and a large gap $\Delta \sim 50$ MeV, as suggested for CFL phase in recent works (Alford et al. 1998; Rapp et al. 1999). Horvath et al. (1991) have discussed the cooling of superconducting QS and QCNS for very small gaps with critical temperatures $T_{c9} = 0.1, 0.5, 1$. However, for QDU processes the suppression factor $\exp(-2\Delta/T)$ has been used rather than $\exp(-\Delta/T)$. The value of the critical temperature $T_{c9} = 0.1$ seems to be quite small, therefore we use $\Delta = 0.1 \dots 1$ MeV for the case of small gaps.

We calculate the cooling history of QS and QCNS using eq. (10), where now summation over j implies summation of emissivities of QDU, eq. (1), suppressed by $\exp(-\Delta/T)$, QMU and QB, eq.(2), suppressed by $\exp(-2\Delta/T)$, emissivity of the crust, eq. (3), and emissivity of photon-gluon decay, eq. (13). Summation over i implies summation of the quark contribution evaluated according to eq. (14), the electron contribution, eq. (6), the massless photon-gluon Goldstone contribution which coincides with that given by eq. (7) and the contributions of massive gluons given by eq. (8) suppressed by $\exp(-m_{\gamma-g}/T)$ and thus being very tiny. The contribution of the crust to the specific heat is negligible. Also in the CFL phase exist 9 hadronic quasi-Goldstone modes. Although their masses are not known we may roughly estimate them as $m_h > m_q$, where m_q is the bare quark mass with a minimum value of ~ 5 MeV. With these masses the contribution of hadronic quasi-Goldstone modes is also very small at temperatures of our interest and can be neglected.

²However, the penetration depth of the external magnetic field is associated with the above mentioned small photon mass rather than with the massive photon-gluon excitation or with the massless Goldstone excitation, see (Blaschke et al. 1999).

The dotted curves in Figs. 1 - 3 demonstrate the cooling history of QCNS and QS for the gap $\Delta = 0.1$ MeV, whereas the dashed lines correspond to the cooling of the CFL phase for $\Delta = 50$ MeV. All thin dotted and dashed lines correspond to the case when the process of massive mixed photon-gluon decay is artificially excluded whereas the corresponding thick lines represent the cooling history when this process is taken into account according to Eq. (13). This new process essentially influences on the early stage of the cooling although the mass of the mixed photon-gluon excitation was supposed to be very high ($m_{\gamma,g} = 70$ MeV for $Y_e = 10^{-5} > Y_{ec}$, lower panel, and $m_{\gamma,g} = 60$ MeV for $Y_e = 0$, upper panel). This is due to a big numerical factor in eq. (13).

In all the cases we obtain very rapid cooling in disagreement with the data. Particularly rapid cooling occurs for the CFL phase. In the latter case contributions of the QDU, QMU and QB processes to the emissivity are suppressed as well as the quark contribution to the specific heat. The rate is governed by the photon emissivity from the surface. For $Y_e > Y_{ec}$, the specific heat is determined by the electrons. As the consequence of this reduction of the specific heat we get a very rapid cooling of the CFL ($Y_e > Y_{ec}$) phase, see lower panel of Figs. 1 - 3. For $Y_e = 0$ (upper panel of Figs. 1 - 3) there are no electrons and the specific heat is determined by a very small contribution of the Goldstone mode given by eq. (7), so that both QCNS and QS cool down even faster than for the case $Y_e = 10^{-5} > Y_{ec}$. In both $Y_e > Y_{ec}$ and $Y_e < Y_{ec}$ cases the cooling time of the CFL phase is extremely small. This means that in reality the cooling is governed by the heat transport in the thin crust (Pizzochero 1991), which we did not take into account.

Thus we see that QCNS and QS, if present among the objects measured in X-rays, can't be in the CFL phase. The cooling of this phase is so rapid that one might expect problems not only for the models of QS and QCNS but also for the models of NS with quark cores consisting of the CFL phase only in deep interiors.

3.2. Cooling of the 2SC phase

This phase is probably more reliable for QCNS rather than for QS since the CFL phase is energetically favorable in the latter case. The 2SC

phase is characterized by large gaps, $\Delta \sim 100$ MeV (Alford et al. 1998; Rapp et al. 1998, 1999). To be specific we suppose that blue-green and green-blue ud -quarks are paired, whereas red u - and d -quarks (u_r, d_r) remain unpaired. This has the consequence that the QDU processes on the red (unpaired) quarks, as $d_r \rightarrow u_r e \bar{\nu}$, as well as QMU, $d_r q_r \rightarrow u_r q_r e \bar{\nu}$, and QB, $q_{1r} q_{2r} \rightarrow q_{1r} q_{2r} \bar{\nu} \nu$, are not blocked by the gaps whereas other processes involving paired quarks are blocked out by large diquark gaps. The QDU process on red quarks occurs in the $Y_e > Y_{ec}$ case only. Its emissivity is given by

$$\epsilon_{\nu}^{\text{QDU}}(d_r) \sim 10^{25} \alpha_s (\rho_b/\rho_0) Y_e^{1/3} T_9^6 \text{ erg cm}^{-3} \text{ sec}^{-1}. \quad (15)$$

The extra suppression factor of the rate (1) comes from the fact that the number of available unpaired color states is reduced.

QMU and QB processes on red quarks are also rather efficient. Although there is no one-gluon exchange between $d_r - d_r$, the QMU and QB processes may go via a residual quark-quark interaction, e.g. via two-gluon exchange. We roughly estimate the corresponding emissivities as

$$\begin{aligned} \epsilon_{\nu}^{\text{QMU}}(d_r q_r) &\sim \epsilon_{\nu}^{\text{QB}}(q_{1r} q_{2r}) \\ &\sim 10^{19} T_9^8 \text{ erg cm}^{-3} \text{ sec}^{-1}. \end{aligned} \quad (16)$$

In the 2SC ($Y_e > Y_{ec}$) phase the QDU process on red quarks is the dominant process and QMU and QB processes on red quarks are subdominant processes whereas in the 2SC ($Y_e < Y_{ec}$) phase QDU processes do not occur and QMU and QB processes on red quarks become the dominant processes. Other processes like QDU, QMU and QB with participation of other color and flavor quarks are continued to be appreciably suppressed by large gaps.

The specific heat is also changed in the 2SC phase since the d_r and u_r contributions are not suppressed by a factor $\exp(-\Delta/T)$ whereas color-paired ud -contributions remain to be suppressed. With these findings we calculate the cooling history of QCNS and QS. The results are presented in Fig. 5 for $Y_e = 10^{-5}$, $\rho = 3\rho_0$ (thick lines), and $Y_e = 0$, $\rho = 5\rho_0$ (thin lines). We see that in both cases the cooling history of QCNS and also of QS with a tiny crust ($T_s = 5 \cdot 10^{-2} T$) nicely agrees with the X-ray data. The cooling of QS with negligible crust does not agree with the data.

4. Conclusions

We have estimated the contributions of various quark processes to the emissivity. Among them, the new decay process of the massive mixed photon-gluon excitation is operating at the early stage of the cooling and QDU, QMU and QB processes on red quarks determine the cooling of the 2SC phase. We discussed the cooling history of QS and QCNS taking into account different possibilities: $Y_e > Y_{ec}$ and $Y_e < Y_{ec}$, the normal quark phase, and various color superconducting phases as the “*uds*-phase - small gaps” suggested by Bailin and Love (1984), the CFL phase, and the 2SC phase, as suggested in recent works (Alford et al. 1998; Rapp et al. 1998; Schäfer 1998; Alford et al. 1999; Rapp et al. 1999). In all the cases we see that *QS and QCNS being in the CFL phase cool down extremely fast in disagreement with known X-ray data*. Also the cooling curves for the case of small gaps ($\Delta = 0.1 \dots 1$ MeV) disagree with the data.

Even if the CFL phase would be realised only in the deep interior region of a NS it would be problematic to satisfy X-ray data. In this case the star would radiate mostly not from the surface but from the CFL region due to its extremely small specific heat related to the Goldstone excitation. Thus the cooling time would be determined by the heat transport from exterior regions to the center rather than by the cooling of the hadronic shell.

The cooling history of the QS and QCNS with a crust being in the normal state agrees with the data.

In this respect the following remark is in order. It is now believed that quark matter below $T_c \sim 50$ MeV is in the color superconducting state characterized by a diquark condensate with large energy gaps ($\Delta \sim 100$ MeV) rather than being in the normal state or the superfluid state characterized by small gaps ($\Delta \lesssim 1$ MeV). If so, one could think that our above discussion of normal quark matter and of the case of a small gap has just pedagogic reasoning. However, this is not really so. Indeed, besides the idea of abnormal strange nuclei and strange stars (Bodmer 1971; Witten 1984; de Rujula and Glashow 1984) there is the very similar idea of abnormal pion condensate nuclei and stars with pion condensate nuclei, see (Migdal 1971; Voskresensky 1977) and the review (Migdal

et al. 1990), chapters 15, 16. The same relates to the kaon condensate objects. Pion condensate systems cool down at about the same rate as given by QDU processes (for $Y_e \sim 10^{-5}$)³. Besides, they can be in the normal state or in the superfluid state characterized by very small gaps $\Delta \lesssim 0.1$ MeV. The cooling history of systems being in normal state is described by thick solid curves on the lower panels of Figs. 1 - 3. Thus we may also conclude that *the hypothesis of pion condensate nuclei-stars (being in normal $\Delta = 0$ state with a crust) does not contradict to the X-ray observations*. Stars with pion and kaon condensate nuclei being in the superfluid state with gaps $\Delta \gtrsim 0.1$ MeV are ruled out as objects being observed in X-rays.

The cooling history of the 2SC phase of QCNS and QS with a tiny crust ($T_s = 5 \cdot 10^{-2}T$) agrees with the X-ray data. The cooling history of QS with no crust disagrees with X-ray data.

Three final remarks are in order:

(i) It is conceivable that there are more complex collective effects which essentially affect the specific heat and the luminosity. E.g., we calculated the mixed photon-gluon spectrum in a simplified model of two Abelian gauge fields and concluded that the mass of the excitation is large, whereas one can't exclude that in the realistic non-Abelian case there exists a photon-gluon excitation of a small mass that could lead to very efficient cooling via the mixed photon-gluon decay process given by eq. (13). The masses of hadronic quasi-Goldstone modes in CFL phase should be carefully studied.

(ii) As we mentioned above in the discussion of the DU process, we have neglected the contribution of strange quarks (QDU-s) relative to that of the light quarks. The former contribution is $\sim 10^{-3}$ times smaller than the latter in the case of normal matter and in the case when all diquark gaps are identical, the discussion given for the CFL phase applies. However, if the pairing gaps for strange diquarks are smaller than those for non-strange diquarks the QDU-s contribution to the emissivity can be essentially enhanced. Up to now

³One should bear in mind a strong suppression of emissivities of pion condensate processes due to nucleon-nucleon correlation effects (Blaschke et al. 1995) and an enhancement of the specific heat (Voskresensky and Senatorov 1984, 1986; Migdal et al. 1990) which are often ignored in the cooling simulations.

there exist only rough estimates of the values of the gaps and this gives rise to large uncertainties in final estimates of the emissivity. Above, in order to be specific, we made calculations considering QDU on u- and d- quarks only. The inclusion of QDU-s in the case when the strange diquark gaps can be smaller than those for non-strange diquarks is simulated by varying the values of the gaps in a wide interval. This does not change the qualitative picture of the QS and QCNS cooling we have discussed in the present work.

(iii) We also would like to point out that, if the compact object formed in the explosion of SN 1987A was a QS or a QCNS being in the CFL phase ($Y_e < Y_{ec}$), it is now so cold that it is already impossible to observe it in soft X-rays. This becomes particularly interesting if continued observation of SN 1987A would find a pulsar and would not observe it in X-rays.

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Fig. 1.— Cooling history of QCNS $M = 1.4 M_\odot$. The relation between T_s and T is given by eq. (5). Lower panel: $Y_e = 10^{-5}$, $\rho = 3\rho_0$, and $\alpha_s = 1$; upper panel: $Y_e = 0$, $\rho = 5\rho_0$, and $\alpha_s = 0.7$. For the thin dotted and dashed lines the process of massive mixed photon-gluon decay is artificially excluded; the corresponding thick lines take this process into account. Line styles are related to different values of the gap Δ , given in the legend. Data points are taken from analysis Schaab et al. (1999).

Fig. 2.— Same as in Fig. 1 but for QS with $T_s = 5 \cdot 10^{-2}T$.

Fig. 3.— Same as in Fig. 1 but for QS with $T_s = T$.

Fig. 4.— Photon- gluon decay via intermediate quark- quark hole and electron- electron hole states.

Fig. 5.— Cooling history of QCNS and QS being in 2SC phase, $M = 1.4 M_\odot$, $\Delta = 100$ MeV. Thick lines: $Y_e = 10^{-5}$, $\rho = 3\rho_0$, and $\alpha_s = 1$; thin lines: $Y_e = 0$, $\rho = 5\rho_0$, and $\alpha_s = 0.7$. Solid curves correspond to QCNS, the relation between T_s and T given by eq. (5), dashed lines correspond to QS with a tiny crust, $T_s = 5 \cdot 10^{-2}T$, and dotted ones to QS without crust, $T_s = T$.

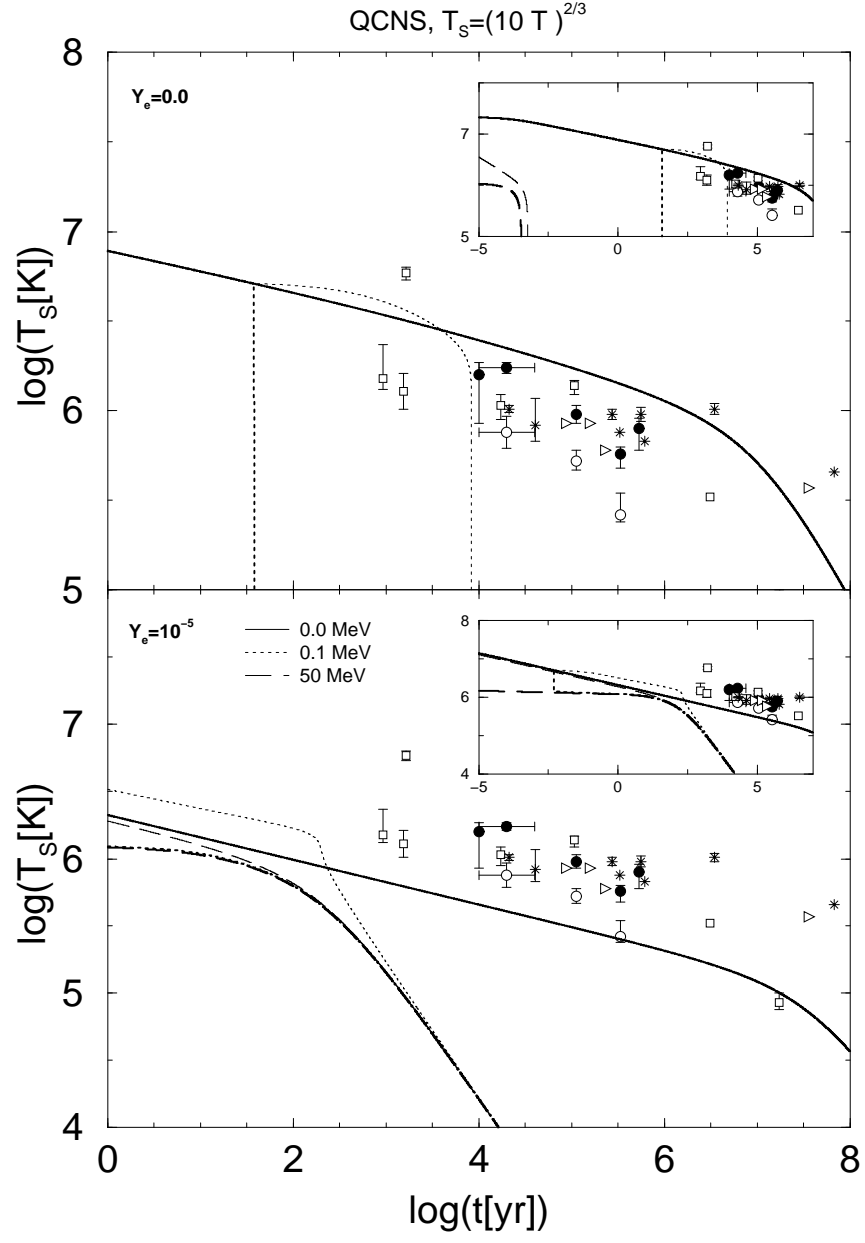


Fig. 1: Blaschke et al.

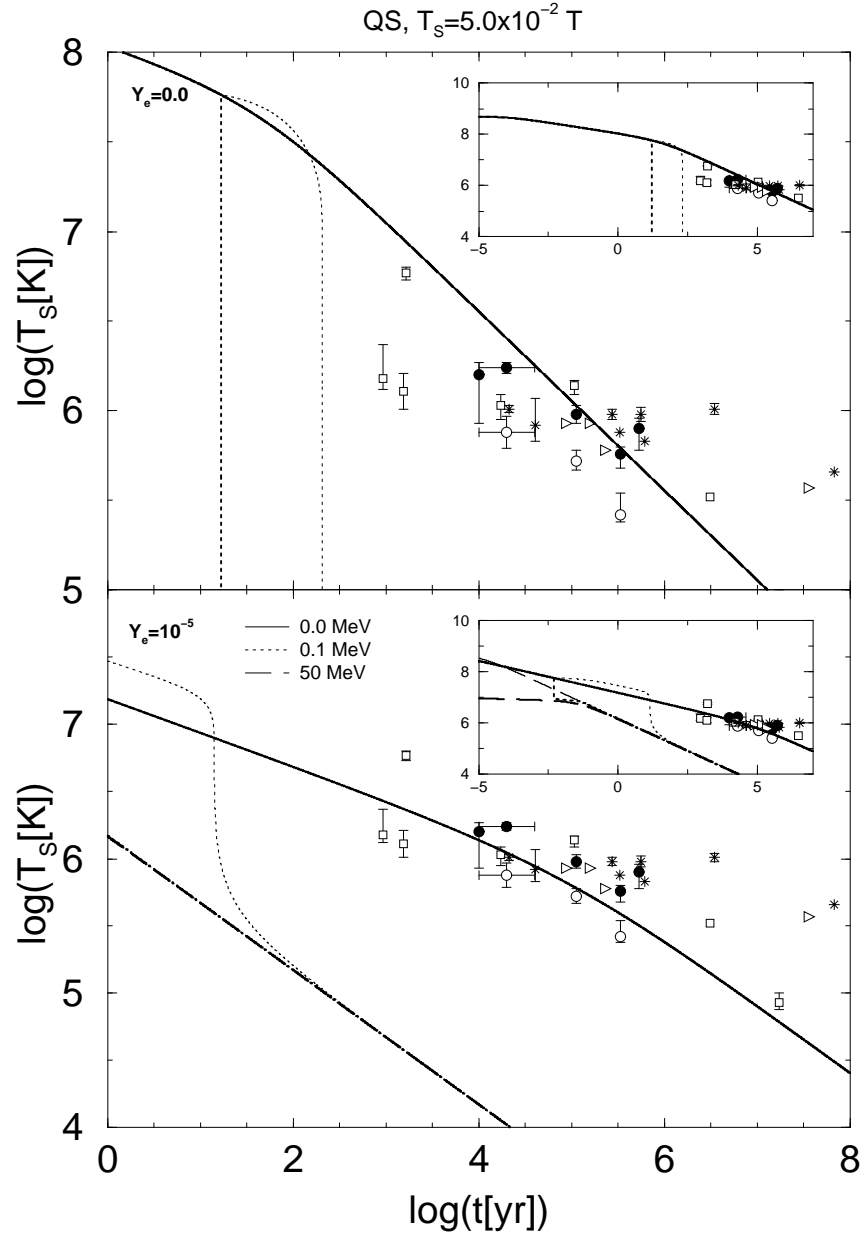


Fig. 2: Blaschke et al.

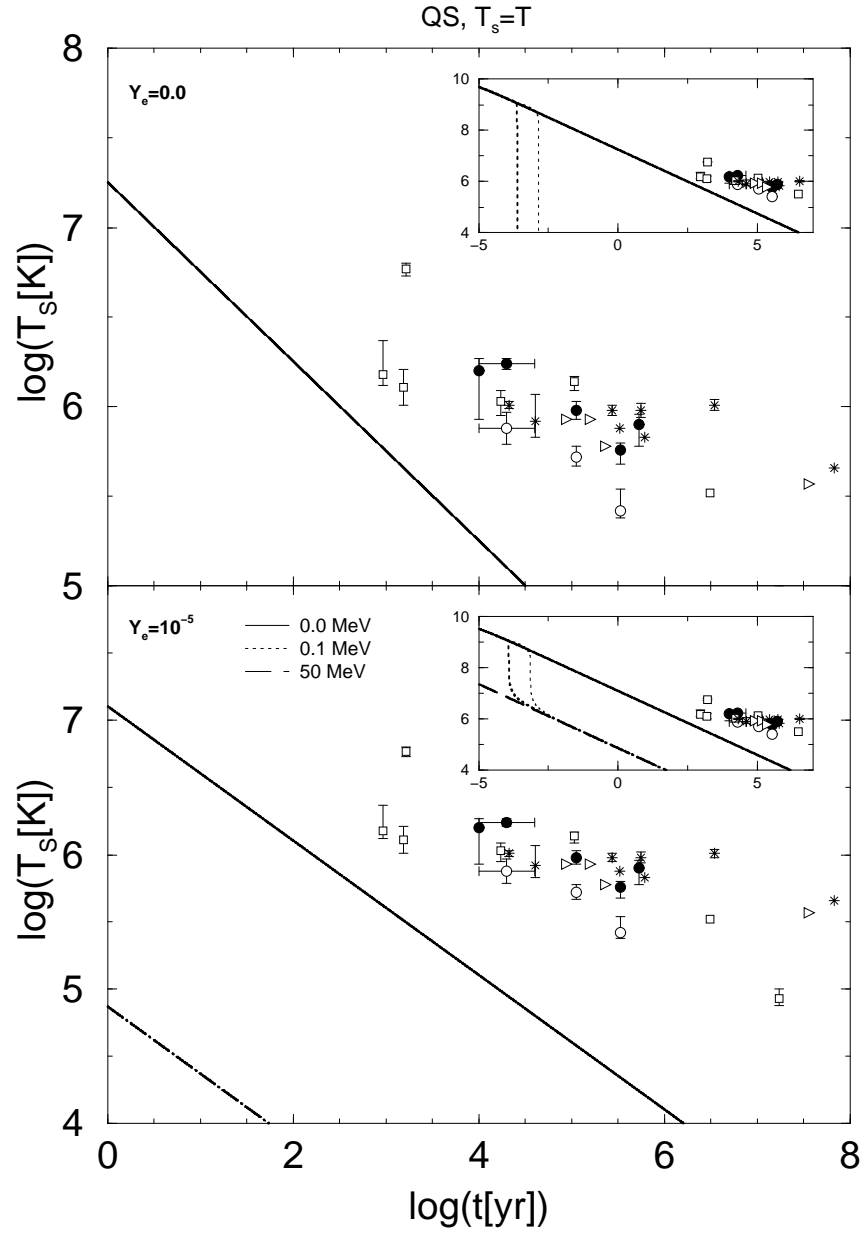


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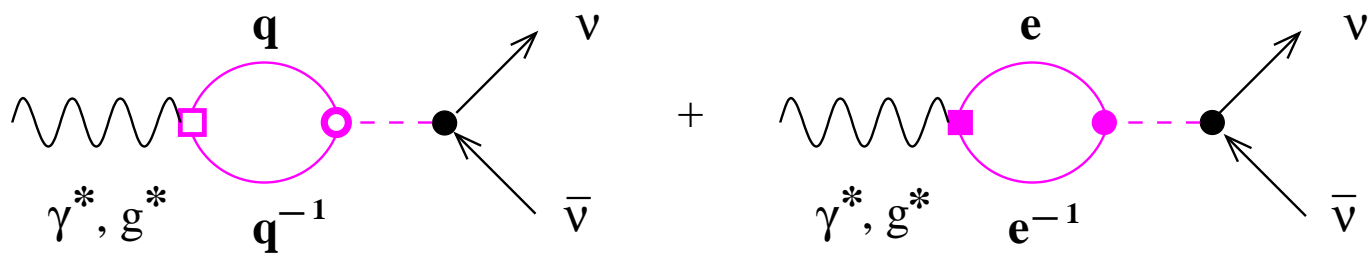


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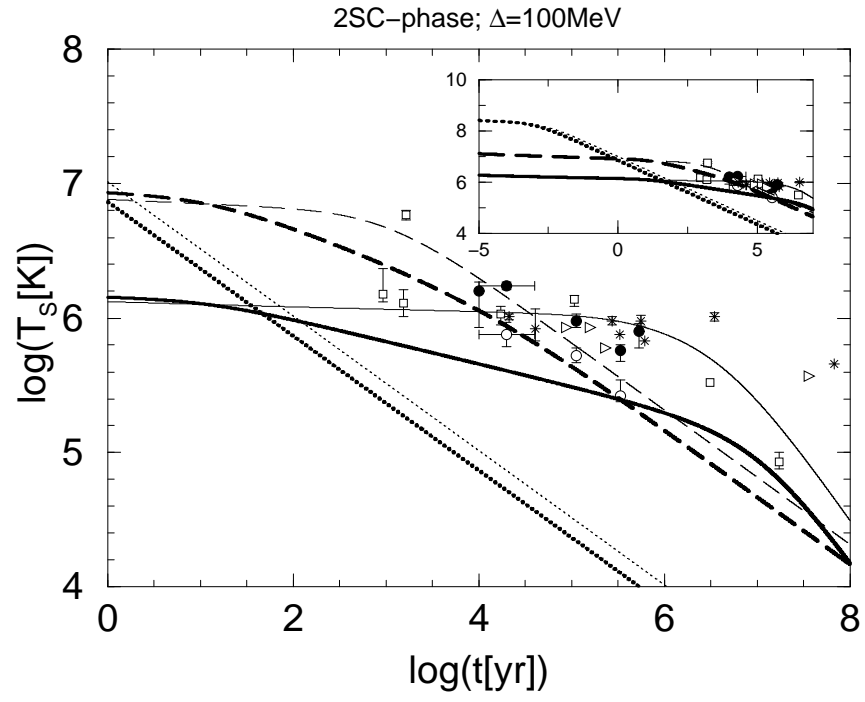


Fig. 5: Blaschke et al.